Dec 2018

MATHEMATICS TEST

60 Minutes—60 Questions

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

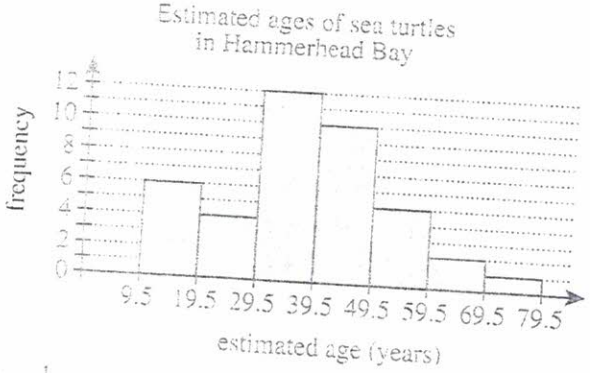
You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

DO YOUR FIGURING HERE.

1. Marine biologists collected a sample of 40 adult sea turtles from Hammerhead Bay and estimated the age, in whole years, of each turtle. The histogram below shows the frequency of sea turtles for each of the estimated age groups. What is the maximum possible number of sea turtles that could have had an estimated age of 45 years?



A. 1
B. 5
C. 9
D. 10
E. 40

2. The sum of the measures of the interior angles of a convex polygon with t sides is $180(t-2)$ degrees. What is the sum of the measures of the interior angles of a convex polygon with 10 sides?

F. 900°
G. 1,440°
H. 1,798°
J. 1,800°
K. 2,160°

DO YOUR FIGURING HERE.

3. In the standard (x,y) coordinate plane, point A has coordinates $(-8,-3)$. Point A is translated 8 units to the right and 3 units up, and that image is labeled A'. What are the coordinates of A' ?

   A. $(-16, -6)$
   B. $(-11,-11)$
   C. $(-8, -6)$
   D. $(0, 0)$
   E. $(16, 6)$

4. For all nonzero values of x and y, which of the following expressions is equivalent to $-\frac{28x^4y^3}{4xy}$?

   F. $-7x^3y^2$
   G. $-7x^4y^4$
   H. $-7x^5y^4$
   J. $-24x^3y^2$
   K. $-32x^3y^2$

5. What is the volume, in cubic inches, of a right circular cone with radius 3 inches and height 6 inches?

   (Note: The volume of a right circular cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.)

   A. 6π
   B. 12π
   C. 18π
   D. 27π
   E. 36π

6. The expression $(x^4)^6$ is equivalent to:

   F. x^{10}
   G. x^{24}
   H. $x^{4.096}$
   J. $6x^2$
   K. $6x^3$

7. Milo earns his regular pay of \$12.00 per hour for up to 40 hours of work per week. For each hour over 40 hours of work per week, Milo earns $1\frac{1}{2}$ times his regular pay. How much does Milo earn in a week in which he works 45 hours?

   A. \$337.50
   B. \$540.00
   C. \$570.00
   D. \$607.50
   E. \$810.00

DO YOUR FIGURING HERE.

8. Which of the following matrices is equal to $\begin{bmatrix} 5 & 7 \\ -4 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 6 & 8 \end{bmatrix}$?

F. $\begin{bmatrix} -1 & 10 \\ 2 & 12 \end{bmatrix}$

G. $\begin{bmatrix} -1 & 10 \\ 10 & 12 \end{bmatrix}$

H. $\begin{bmatrix} 11 & 10 \\ 10 & 12 \end{bmatrix}$

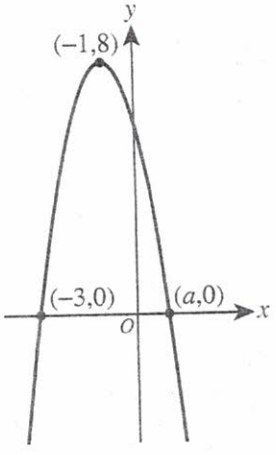
J. $\begin{bmatrix} 12 & -3 \\ 0 & 14 \end{bmatrix}$

K. $\begin{bmatrix} 12 & 71 \\ 48 & 20 \end{bmatrix}$

9. Tomi has 6 pairs of shoes, 4 pairs of pants, and 6 shirts, which can be worn in any combination. He needs to choose a clothes combination to wear to the school dance. How many different combinations consisting of 1 of his 6 pairs of shoes, 1 of his 4 pairs of pants, and 1 of his 6 shirts are possible for Tomi to wear to the dance?

A. 10
B. 16
C. 24
D. 48
E. 144

10. In the standard (x,y) coordinate plane below, the graph of the equation $y = -2(x + 1)^2 + 8$ intersects the x-axis at points $(-3,0)$ and $(a,0)$ and has its vertex at point $(-1,8)$. What is the value of a?



F. $\frac{1}{2}$

G. 1

H. $\frac{3}{2}$

J. 2

K. 3

11. What is the least common denominator of the fractions $\frac{4}{21}$, $\frac{1}{24}$, and $\frac{3}{16}$?

A. 112
B. 336
C. 504
D. 2,688
E. 8,064

DO YOUR FIGURING HERE.

12. In the standard (x,y) coordinate plane, what is the slope of the line through $(-7,3)$ and $(2,4)$?

F. $-\frac{7}{5}$

G. $-\frac{1}{5}$

H. $-\frac{1}{9}$

J. $\frac{1}{9}$

K. $\frac{1}{5}$

13. A group of 60 students and 4 sponsors took a field trip to a local museum. For their first guided tour, students were given a choice of 1 of 3 art exhibits. Of the 60 students, $\frac{1}{2}$ chose Modern, $\frac{1}{4}$ chose American Folk, and $\frac{1}{6}$ chose Western. Each student that expressed a choice chose exactly 1 exhibit. The remaining students expressed no choice. How many of the students expressed no choice?

A. 5
B. 6
C. 10
D. 15
E. 30

14. What is the greatest integer solution to $6x - 2 \leq 11.2$?

F. -2
G. -1
H. 1
J. 2
K. 3

15. Classics Online charges a onetime registration fee of \$17.50 and sells classical music downloads for \$0.70 per song. Ian has \$50.00 that he will use to pay the registration fee and buy classical music from Classics Online. What is the maximum number of songs Ian can buy?

A. 25
B. 32
C. 35
D. 46
E. 53

DO YOUR FIGURING HERE.

16. The lengths of corresponding sides of 2 similar right triangles are in the ratio 2:9. The hypotenuse of the smaller triangle is 8 inches long. How many inches long is the hypotenuse of the larger triangle?

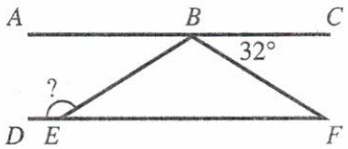
F. 4.5
G. 11
H. 15
J. 18
K. 36

17. The mean age of the 5 people in a room is 30 years. One of the 5 people, whose age is 50 years, leaves the room. What is the mean age of the 4 people remaining in the room?

A. 14
B. 20
C. 25
D. 30
E. 35

18. In the figure below, B lies on $\overline{AC}$, E lies on $\overline{DF}$, $\overline{AC} \parallel \overline{DF}$, $\triangle EBF$ is isosceles with $\overline{BE} \cong \overline{BF}$, and $\angle CBF$ measures 32°. What is the measure of $\angle BED$?

F. 106°
G. 116°
H. 122°
J. 132°
K. 148°



19. If $x = -1$ and $y = 2$, what is the value of $x^3 - 2x^2y - 4xy^2 + 8$?

A. −13
B. −5
C. 19
D. 23
E. 27

20. A retailer is comparing the costs of buying 3 products from 1 of 2 companies. The cost of each product from both companies is shown in the table below.

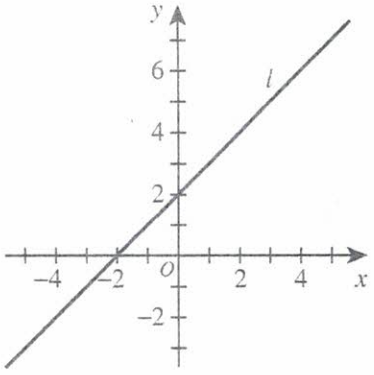
	Company A	Company B
Product 1	\$ 5.00	\$5.50
Product 2	\$10.00	\$9.50
Product 3	\$ 6.00	\$5.75

The retailer will buy 10 of Product 1, 12 of Product 2, and 5 of Product 3. How much more will it cost the retailer to purchase these products from Company A than to purchase these products from Company B ?

F. \$ 0.25
G. \$ 1.25
H. \$ 2.25
J. \$ 6.75
K. \$12.25

DO YOUR FIGURING HERE.

21. Line l is shown in the standard (x,y) coordinate plane below. Alicia drew line p with a slope that is $\frac{1}{2}$ of the slope of line l and with a y-intercept that is 3 times the y-intercept of line l. One of the following equations represents line p. Which one?



A. $y = \quad x + 3$

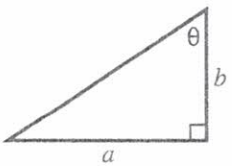
B. $y = -3x + 1$

C. $y = \quad 3x + 1$

D. $y = -\frac{1}{2}x + 6$

E. $y = \frac{1}{2}x + 6$

22. One angle measure and 2 side lengths, in inches, are given in the right scalene triangle below. In terms of a and b, what is $\sin\theta$?



F. $\frac{a}{b}$

G. $\frac{b}{a}$

H. $\frac{a}{\sqrt{a^2+b^2}}$

J. $\frac{b}{\sqrt{a^2+b^2}}$

K. $\frac{\sqrt{a^2+b^2}}{b}$

23. Each side of square $ABCD$ has a length of 50 cm. A certain rectangle whose area is equal to the area of $ABCD$ has a width of 10 cm. What is the length, in centimeters, of the rectangle?

A. 40
B. 50
C. 60
D. 125
E. 250

24. If $f(x) = 5x^2 - 6x + 1$ and $g(x) = x^2 - 2$, which of the following expressions represents $(fg)(x)$?

F. $6x^2 - \quad 6x - 1$

G. $5x^4 - 26x^2 + 33$

H. $5x^4 - 20x^2 - 6x + 21$

J. $5x^4 - \quad 6x^3 - 9x^2 + 12x - 2$

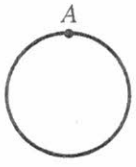
K. $25x^4 - 60x^3 + 46x^2 - 12x - 1$

DO YOUR FIGURING HERE.

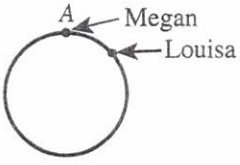
25. A bag contains 12 red marbles, 14 yellow marbles, and 8 green marbles. How many additional red marbles must be added to the 34 marbles already in the bag so that the probability of randomly drawing a red marble is $\frac{3}{5}$?

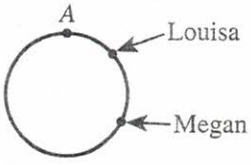
A. 16
B. 21
C. 29
D. 34
E. 44

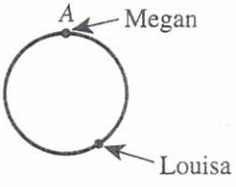
26. Megan and Louisa are side-by-side (at point A in the figure shown below) when they begin to run at the same time clockwise (C) around a small circular track. Megan runs at a rate of 40 seconds per lap, while Louisa runs at a rate of 70 seconds per lap.

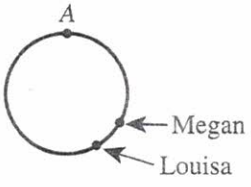


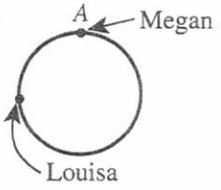
Which of the following figures best represents Megan's and Louisa's locations 120 seconds after they begin to run?

F.


J.


G.


K.


H.


27. The statement $3x - (x + 6) + 8 = 2x + 14$ is true for:

A. $x = 0$ only.
B. $x = 4$ only.
C. $x = 7$ only.
D. all values of x.
E. no values of x.

DO YOUR FIGURING HERE.

28. The first 7 terms in an arithmetic sequence are listed below. What is the difference between the mean and the median of the 7 terms?

$$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$$

F. 0

G. $\frac{3}{7}$

H. $\frac{1}{2}$

J. 2

K. 9

29. What is the product of the complex numbers $(-2i + 5)$ and $(2i + 5)$?

A. 3
B. 21
C. 29
D. $20i - 21$
E. $20i + 21$

30. Which of the following expressions represents the sum of 3.8×10^5 and 6.4×10^4 in scientific notation?

F. 1.02×10^{10}
G. 4.44×10^4
H. 4.44×10^5
J. 10.2×10^{20}
K. 44.4×10^4

31. Lian has $6\frac{1}{2}$ yards of ribbon she will use to make bows. She will use $\frac{3}{4}$ yard of ribbon to make each bow. After Lian has made all the bows possible with the ribbon, what length of ribbon, in yards, will NOT have been used to make bows?

A. 0

B. $\frac{1}{2}$

C. $\frac{21}{32}$

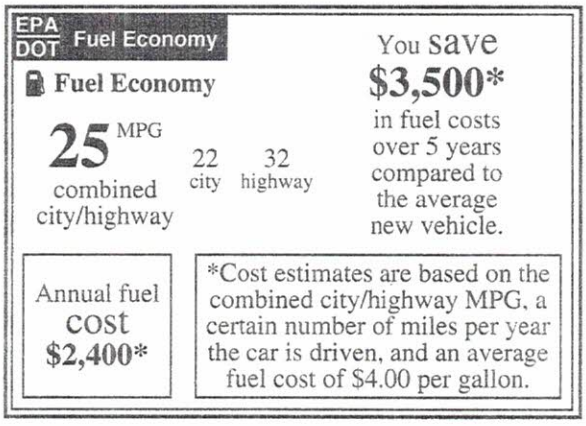
D. $\frac{2}{3}$

E. $\frac{7}{8}$

DO YOUR FIGURING HERE.

Use the following information to answer questions 32–34.

Carl purchased a new car. The fuel economy window sticker on the new car contained the information shown below. In this figure, MPG is miles per gallon.



32. Carl is planning a trip in his new car that will include 350 miles of highway driving. Using the average fuel cost per gallon given in the fuel economy window sticker, which of the following dollar amounts is closest to his total cost for fuel over the 350 miles of highway driving?

F. $43.75
G. $51.85
H. $56.00
J. $63.64
K. $87.50

33. The cost estimates are based on a certain number of miles driven per year. To the nearest 1,000 miles, what is this number?

A. 13,000
B. 15,000
C. 16,000
D. 19,000
E. 22,000

34. Based on the annual fuel cost estimate for this car and the estimate for how much Carl will save in fuel costs over the next 5 years, what would be the expected annual fuel cost of an average new vehicle?

F. $1,180
G. $2,950
H. $3,100
J. $3,500
K. $3,980

DO YOUR FIGURING HERE.

35. A certain race car has a maximum speed of 240 miles per hour. Which of the following is an expression for this maximum speed in feet per second?

(Note: 1 mile = 5,280 feet)

A. $\frac{240(5{,}280)}{36{,}000}$

B. $\frac{240(3{,}600)}{5{,}280}$

C. $\frac{240(5{,}280)}{3{,}600}$

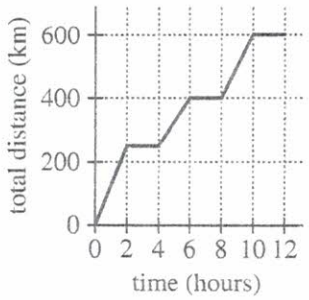
D. $\frac{60(5{,}280)}{240}$

E. $\frac{240(5{,}280)}{360}$

36. A chemist needs 1 ounce of element X. The only way which the chemist can get element X is to buy compound Y, which contains 10% X. Compound Y costs \$2.40 per pound (16 ounces). How much must the chemist pay in order to ensure that she receives 1 ounce of element X ?

F. \$ .15
G. \$ .24
H. \$1.50
J. \$2.40
K. \$3.84

37. Emi traveled to 3 locations during a workday. Emi remained at each location a whole number of hours. The graph below shows the relationship between time, in hours, into her workday and total distance, in kilometers, traveled. Which of the following values is closest to Emi's average speed, in kilometers per hour, for the parts of the workday when she was traveling?



A. 60
B. 75
C. 80
D. 100
E. 125

38. What are all and only the values of x that are NOT in the domain of the function $f(x) = \frac{(x-7)(x+2)}{(x+6)(x-8)}$?

F. −8 and 6
G. −6 and 8
H. −2 and 7
J. −8, −7, 2, and 6
K. −6, −2, 7, and 8

DO YOUR FIGURING HERE.

39. A new band asked its audience to rate the band's performance on a scale from 1 (poor) through 5 (excellent). The table below gives the percentage of the audience that gave each of the ratings. To the nearest 0.1, what was the mean rating given by this audience?

Rating	Percentage
1	0%
2	0%
3	10%
4	70%
5	20%

A. 2.0
B. 2.8
C. 3.0
D. 4.0
E. 4.1

40. In $\triangle DEF$, the length of $\overline{DE}$ is $\sqrt{60}$ cm, and the length of $\overline{EF}$ is 6 cm. If it can be determined, what is the length, in centimeters, of $\overline{DF}$?

F. 6
G. $\sqrt{60}$
H. $\sqrt{66}$
J. $\sqrt{96}$
K. Cannot be determined from the given information

41. Let a and b represent real numbers with the property $|a - b - 1| > 0$. Which of the following statements about a and b CANNOT be true?

A. $a - b < 1$
B. $a - b = 1$
C. $a < 1$ and $b > 0$
D. $a < 1$ and $b = 0$
E. $a < 0$ and $b > 0$

42. Juwan has 150 cm of wire. For a craft project, he uses all the wire to make 1 circle with a radius of 5 cm and 1 square. To the nearest 0.1 cm, what is the side length of the square?

F. 17.9
G. 29.6
H. 32.6
J. 33.6
K. 118.6

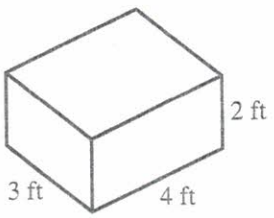
DO YOUR FIGURING HERE.

Use the following information to answer questions 43–46.

A local recycling center pays customers for cans, bottles, and cardboard. After processing these items, the center then resells them to XYZ Inc. The recycling center's payments to customers and the resale prices paid by XYZ Inc. are given in the table below.

Item	Payment	Resale price
Can	\$0.05 each	\$0.15 each
Bottle	\$0.10 each	\$0.18 each
Cardboard	\$0.01 per pound	\$0.02 per pound

43. The closed box with no overlapping pieces whose dimensions are given below is made with cardboard that weighs 1 pound per square foot of the surface area of the box. Which of the following values is closest to the amount the recycling center will pay a customer for this box?



A. \$0.24
B. \$0.36
C. \$0.48
D. \$0.52
E. \$1.04

44. The cost of processing cans and bottles at the recycling center is \$0.03 per can and \$0.02 per bottle. After paying the processing cost and the payment to customers, what is the recycling center's profit on the resale of 200 cans and 300 bottles to XYZ Inc. ?

F. \$ 32.00
G. \$ 40.00
H. \$ 48.00
J. \$ 84.00
K. \$112.00

45. To the nearest 1%, the recycling center's payment to a customer for a bottle is what percent of the resale price of a bottle sold to XYZ Inc. ?

A. 8%
B. 56%
C. 80%
D. 125%
E. 180%

46. In 1 shipment, the recycling center sold a total of 2,700 cans and bottles to XYZ Inc. for \$441.00. How many bottles were in the shipment?

F. 1,200
G. 1,230
H. 1,350
J. 1,470
K. 1,500

DO YOUR FIGURING HERE.

47. Given constants c, d, m, and n such that $x^2 + mx + c$ has factors of $(x + 2)$ and $(x + 4)$ and $x^2 + nx + d$ has factors of $(x + 3)$ and $(x + 7)$, what is mn ?

A. 16
B. 18
C. 29
D. 60
E. 168

48. For every angle θ, measured in radians, which of the following is equal to $\sin(2\pi + \theta)$?

F. $\sin(-\pi + \theta)$

G. $\sin(\theta)$

H. $\sin\left(\frac{\pi}{2} + \theta\right)$

J. $\sin\left(\frac{2\pi}{3} + \theta\right)$

K. $\sin(\pi + \theta)$

49. A small circle and a large circle are tangent at T, as shown in the figure below. The center, O, of the large circle lies on the small circle. The diameter of the large circle is 12 cm. What is the ratio of the area of the small circle to the area of the large circle?

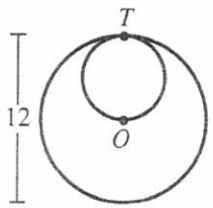
A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{2}$



50. For all positive values of a and b, which of the following expressions is equal to $\frac{2a}{b} + \frac{b}{2a}$?

F. $\frac{2a + b}{b + 2a}$

G. $\frac{2a + b}{2ab}$

H. $\frac{4a + b}{2ab}$

J. $\frac{4a^2 + b^2}{2ab}$

K. $\frac{4a^2 + b^2}{2a + b}$

DO YOUR FIGURING HERE.

51. The vector **i** represents 1 mile per hour east, and the vector **j** represents 1 mile per hour north. According to her GPS, at a particular instant, Tia is biking 30° west of north at 16 miles per hour. One of the following vectors represents Tia's velocity, in miles per hour, at that instant. Which one?

A. $-8\mathbf{i} - 8\sqrt{3}\mathbf{j}$
B. $-8\mathbf{i} + 8\sqrt{3}\mathbf{j}$
C. $8\mathbf{i} + 8\sqrt{3}\mathbf{j}$
D. $8\sqrt{3}\mathbf{i} - 8\mathbf{j}$
E. $8\sqrt{3}\mathbf{i} + 8\mathbf{j}$

52. Let $(f \circ g)(x) = \sqrt[3]{x+1} - 2$ and $g(x) = x + 1$. Which of the following expressions defines $(g \circ f)(x)$?

F. $\sqrt[3]{x-1}$
G. $\sqrt[3]{x} - 1$
H. $\sqrt[3]{x} - 2$
J. $(x+2)^3 - 1$
K. $(x-1)^3 + 2$

53. Consider sets A, B, C, and D such that B is a subset of A, C is a subset of B, and D is a subset of C. Whenever x is an element of B, x *must* be an element of:

A. A.
B. D.
C. A and C.
D. C and D.
E. A, C, and D.

54. For all positive values of x, which of the following expressions is equivalent to $\sqrt[6]{x^4}\left(\sqrt[3]{x^4}\right)$?

F. $x^{\frac{8}{9}}$
G. x
H. $x^{\frac{9}{8}}$
J. x^2
K. $x^{\frac{9}{4}}$

DO YOUR FIGURING HERE.

55. A company sells ice cream in 2-quart containers for \$3.00 per container. The company also sells ice cream in 1.5-quart containers for \$2.50 per container. What is the ratio of the price per quart for the 2-quart container to the price per quart for the 1.5-quart container?

A. $\frac{9}{10}$

B. $\frac{10}{9}$

C. $\frac{3}{2}$

D. $\frac{5}{3}$

E. $\frac{5}{2}$

56. Each of 100 distinct playing cards is 1 of 5 solid colors and is numbered with 1 integer. There are 20 each of blue, red, yellow, green, and orange cards numbered 1–20. One of the 100 cards will be selected at random. What is the probability that the selected card will be blue OR numbered 17 ?

F. $\frac{5}{100}$

G. $\frac{17}{100}$

H. $\frac{20}{100}$

J. $\frac{24}{100}$

K. $\frac{25}{100}$

57. The changes in a city's population from one decade to the next decade for 3 consecutive decades were a 20% increase, a 30% increase, and a 20% decrease. About what percent was the increase in the city's population over the 3 decades?

A. 10%
B. 20%
C. 25%
D. 30%
E. 70%

58. Four golfers will be randomly split into 2 groups of 2 for a tournament. If Jill and Ramona are among the 4, what is the probability that they will be paired together?

F. $\frac{1}{12}$

G. $\frac{1}{8}$

H. $\frac{1}{6}$

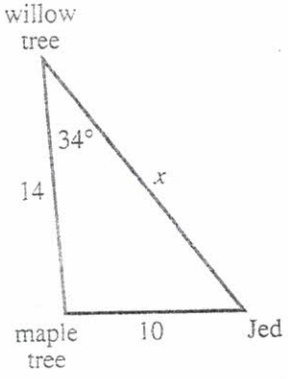
J. $\frac{1}{4}$

K. $\frac{1}{3}$

DO YOUR FIGURING HERE.

59. Jed is standing 10 meters from a maple tree that is 14 meters from a willow tree, as shown in the figure below, in which the measure of an angle is given. Which of the following equations, when solved for x, gives the distance, x meters, between Jed and the willow tree?

(Note: For a triangle with sides of length a, b, and c that are opposite angles $\angle A$, $\angle B$, and $\angle C$, respectively, $c^2 = a^2 + b^2 - 2ab \cos \angle C$.)



A. $10^2 = x^2 + 14^2 - 2x(14)(\cos 34°)$
B. $10^2 = x^2 + 14^2 - 2(10)(14)(\cos 34°)$
C. $14^2 = x^2 + 10^2 - 2x(14)(\cos 34°)$
D. $x^2 = 10^2 + 14^2 - 2x(14)(\cos 34°)$
E. $x^2 = 10^2 + 14^2 - 2(10)(14)(\cos 34°)$

60. Suppose the equations $(x-4)^2 + (y-3)^2 = 4$ and $\frac{(x-4)^2}{4} + \frac{(y-10)^2}{16} = 1$ are graphed in the same standard (x,y) coordinate plane. How many points of intersection do these graphs share?

F. 0
G. 1
H. 2
J. 3
K. 4

END OF TEST 2

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.
DO NOT RETURN TO THE PREVIOUS TEST.